

MATH 118: Final

Name: _____

Directions:

- * Show your thought process (commonly said as "show your work") when solving each problem for full credit.
- * If you do not know how to solve a problem, try your best and/or explain in English what you would do.
- * Good luck!

Problem	Score	Points
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
11		10
12		10

120

1. Short answer questions:

(a) Suppose you try distributing

$$(x + y)^2 z^2 = (xz^2 + yz^2)^2$$

Why is this incorrect?

(b) Suppose you cancel out the x 's to simplify

$$\frac{3 + x}{x} = \frac{3 + 1}{1} = 4$$

Why is this incorrect?

(c) You try simplifying by distributing:

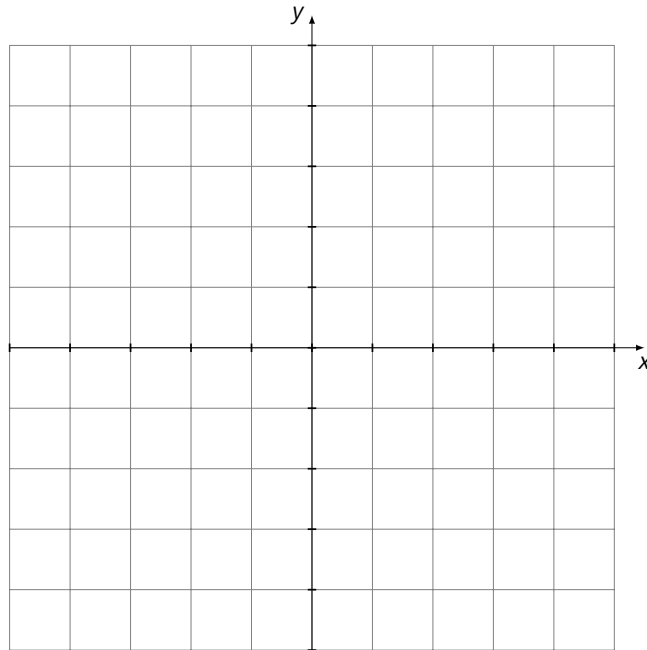
$$[(x - 1)^2 + (x + 1)]^3 = (x - 1)^5 + (x + 1)^3 = x^5 - 1^5 + x^3 + 1^3 = x^5 + x^3$$

Circle the two types of mistakes you made and explain why they are mistakes.

2. Suppose

$$f(x) = \begin{cases} x - 1 & x < 1 \\ 2^x & x \geq 1 \end{cases}$$

(a) Sketch a graph of $f(x)$.



(b) What is $f(1)$?

3. Isolate the variable in the following equations:

(a) $4x + 2 = 6x - w$, for x

(b) $\frac{2x + 1}{x - 2} = 1$, for x .

(c) $4x(y - 1) - 6x(z + 2) - w(4xz - y + 1) = x$, for x

4. Suppose $f(x) = x - x^2$

(a) A person tries to find $f(x + h)$ by writing

$$f(x + h) = x - x^2 + h$$

This is wrong. What expression (involving $f(x)$) did the person actually write down?

(b) The person then tries again:

$$f(x + h) = x + h - x + h^2$$

Explain the reason why this is also incorrect.

(c) Your turn: Evaluate $f(x + h)$ and fully simplify.

(d) In general, when you are substituting two or more terms into **(a)** a variable with a power or **(b)** that variable being subtracted, what do you need to not forget?

5. Solve the equation for x . Check your work if necessary.

(a) $x^2 = 3(x - 1)$

(b) $\frac{1}{x-1} - \frac{2}{x^2} = 0$

6. Solve the equation for x . Check your work if necessary.

(a) $x^2e^x + xe^x - e^x = 0$

(b) $2 \log x = \log 2 + \log(3x - 4)$

7. Fully simplify each expression; **your answer should be a number**. No fractional expressions or negative exponents.

(a) $\log_2 2$

(b) e^0

(c) $8^{2/3}$

(d) $\log_{10} 2 + \log_{10} 5$

(e) $\frac{5}{12} + \frac{13}{16}$

(f) $(1 + i)(1 - i)$

(g) i^4

(h) -2^4

(i) $2^{-5} \cdot 2^4 \cdot \frac{1}{2^3}$

8. Completely factor each expression. Try to obtain a complete factorization.

(a) $16a^2 - 24a + 9$

(b) $x^3 + 4x^2 + x + 4$

(c) $(x + y)^2 - 2(x + y) + 1$

(d) $(x - 1)(x + 2)^2 - (x - 1)^2(x + 2)$

(e) $x^5 - x$

9. Find a complete factorization of

$$f(x) = x^4 - x^3 - x - 1$$

given that $x = i$ and $x = -i$ are zeros. You must use the division algorithm.

10. **Multi-Level Marketing (Exponential Growth)** Suppose your friend is trying to convince you to join a MLM network to sell products. Here are the conditions to join:

- * **Membership:** You must pay USD \$120 yearly to stay as a member. This fee is meant "to cover" the network giving you products to sell.
- * **Compensation plan:** You will not get paid for selling products **until** you recruit 3 or more members. After that, you will only get 15% of your recruits' profits.

For the rest of this problem, assume every member in this network recruits **only three people**.

(a) This network began with one person recruiting three people, then those three people recruiting three more for 9, and so on. We can represent this relationship with a function $f(x)$ where x is the "generation" you were recruited at and $f(x)$ represents the total number of people in generation x . For example,

- i. $f(1) = 1$ because first generation only had one person
- ii. $f(2) = 3$ since three people were recruited
- iii. $f(3) = 9$ since those last three people recruited 3 people only.

Find an explicit formula for $f(x)$ representing an arbitrary generation x . Justify why your formula is correct.

(b) Suppose this network is only taking place in San Luis Obispo, where for the sake of assumption there is 364 **total** people. How many generations will it take for everyone in SLO to be recruited if you're only allowed to recruit from SLO?

(c) Based on part (b), how many people are in the final generation?

(d) Based on your answer in part (b) and (c), what percent of people in SLO are in the final generation?

(e) Explain why the percentage in part (c) shows that the majority of multi-level marketing members lose out on any profits.

(f) Armed with this information, if you were to join a MLM, what do you need to watch out for?

11. Answer the following:

(a) One recurring theme throughout this quarter is given a problem like $x^2 = c$, the two answers that solve this equation are $x = \pm\sqrt{c}$. In English, explain why these two solutions are valid for the equation.

(b) Given the function $g(x) = -\log(-2x + 4)$, describe **in English or with rules** the transformations needed to start from $f(x) = \log(x)$ to get to $g(x)$.

(c) Determine if each expression is a factor of $x^4 - 2x^3 - 2x^2 + 5x - 2$:

i. $x - 2$

ii. $x + 2$

iii. $x - 1$

iv. $x + 1$

(d) Find the leading term and end behavior of $f(x) = 2(x - 3)^2(x + 4)^3$.

(e) If $f(x)$ is one-to-one, can it be increasing on $(-\infty, 2)$ and decreasing on $(2, \infty)$?

(f) Sketch the graph of a function with two local maxima, two local minima, and 4 x -intercepts.

(g) If $f(x) = x + 1$ and $g(x) = x^2 - 1$, find and **fully expand** $-f(x)g(x)$.

(h) Write down a formula of a quadratic that has vertex $(-2, 3)$.

(i) Write down a degree 5 polynomial with four distinct zeroes.

12. Answer the following. Do not leave negative exponents.

(a) Simplify

$$\frac{1}{x-1} + \frac{1}{x+1} - \frac{2}{x}$$

(b) Simplify

$$x^2y \left(\frac{x+1}{y} \right)^{-2} \left(\sqrt{\frac{x}{y}} \right)^4$$

(c) If

$$f(x) = x^2 + 1 \quad g(x) = 2x^3 \quad h(x) = 2x - 1 \quad k(x) = 6x^2$$

fully expand and simplify the following expressions:

i. $f(x)g(x) + h(x)k(x)$

ii. $\frac{g(x)f(x) - k(x)h(x)}{[k(x)]^2}$